Welcome to a new series which will feature some of my shortest routines and programs (usually coded for the HP-71B but occasionally also for other models such as the HP-15C, 34C, 41C, etC.), where the accent is placed on lightness, both lightness of content and lightness of exposition (<= 1 page).

Let’s begin with the topic of prime counting, i.e., finding out how many prime numbers there are up to a given limit N. For large N, generating all primes up to N and returning the count is prohibitive. In fact, getting the exact count for $N > 10^{20}$ is a daunting task requiring utmost computational power. But if we content ourselves with an asymptotic approximation, where “asymptotic” means the larger is N the smaller is the relative error, then this 8-liner is a fast, extremely accurate one:

```
100 DEF FNZ(Z) @ IF Z=2 THEN FNZ=PI*PI/6 ELSE IF Z=3 THEN FNZ=1.20205690316
110 IF Z=4 THEN FNZ=1.0823232371 ELSE IF Z=5 THEN FNZ=1.03692775514
120 IF Z=6 THEN FNZ=1.01734306198 ELSE IF Z=7 THEN FNZ=1.00834927738
130 IF Z<8 THEN END ELSE S=1 @ T=0 @ N=2
140 S=S+N^(-Z) @ N=N+1 @ IF S<>T THEN T=S @ GOTO 140 ELSE FNZ=S
```

This code implements two multiline user-defined functions, namely:

- **FNR(N)** gives a very close approximation to the number of primes up to N
- **FNZ(N)** auxiliary: returns Riemann’s Z function for integer N>1, fast

Let’s test our function **FNR(N)** for $N = 10^3, 10^6, 10^9, 10^{12}, 4.10^{16}$:

```
> FOR I=3 TO 12 STEP 3 @ DISP 10^I,FNR(10^I) @ NEXT I @ DISP 4E16,FNR(4E16)
```

<table>
<thead>
<tr>
<th>Approximation</th>
<th># Primes up to N=1000</th>
<th># Primes up to N = 10^{12}</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/LN(N)</td>
<td>145 (-13.69%)</td>
<td>36191206825 (-3.767040%)</td>
</tr>
<tr>
<td>Log Integral Li(N)</td>
<td>177 ( 5.36%)</td>
<td>37607950280 ( 0.000102%)</td>
</tr>
</tbody>
</table>

As you can see, this is very close to the exact values and the relative error (which never is that big anyway) decreases very quickly for large N. How well does it fare against other well-known approximations? Let’s check for N large and small:

Pretty good, isn’t it? And lots faster than generating and counting primes! ...